Roll No-12

M.sc. 3rd semester

Date of Assignment-21/12/2020

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**Experiment No -11**

**Topic**-**-** Fitting of Auto Regression (AR) series to time series data.

**Problem** –**-** Fit a auto regressive series of order 2 from the following data.



|  |  |
| --- | --- |
| t | ut |
| 1 | 7 |
| 2 | 6 |
| 3 | -6 |
| 4 | -4 |
| 5 | 3 |
| 6 | -4 |
| 7 | -5 |
| 8 | -8 |
| 9 | 10 |
| 10 | 10 |
| 11 | 6 |
| 12 | -4 |
| 13 | -4 |
| 14 | -7 |
| 15 | -2 |
| 16 | 6 |
| 17 | 17 |
| 18 | 24 |
| 19 | 13 |
| 20 | 1 |
| 21 | 1 |

Also find the period of the auto regressive series of order 2.

**Theory and Calculation**-

Fitting of AR(2)

The auto regressive series of order k is given by –



Now, taking the deviation of the observations ut from their mean and replacing the ut’s with these deviations, the series becomes



When, k=2, the AR series of order k reduces to

 -------(1)

Multiplying eqn by  , we get

 ---------(2)

Taking expectation on both sides of equation (2) we get-



 -------(3)

Where,  represents the auto-covariance function at lag k. Dividing both sides by  , we get from (3)

 --------(4)

Where  represents the autocorrelation at lag k.

**[** When k=3, the AR reduces to

 ------(5)

Multiplying both sides of eqn (5) by  and taking expectation, we get—



 -----(6)

Dividing both sides of eqn (6) by  , we get—



 ------(7) **]**

In practice,  is determined by the sample counter part of  i.e. . Putting k = 1 and k = 2 in eqn (4) we get-



And



Replacing  &  by  and  respectively, we get -







 -------(8)

Solving (8) , we get the estimate of the coefficients  ‘s . Therefore the fitted AR series of order 2 is-



The period of the fitted second order AR series is given by-



Finally, we have -





**Calculation-**

we construct the following table-

|  |  |
| --- | --- |
| t | ut |
|  |  |
| 1 | 0 |
| 2 | 0 |
| 3 | 4.66925 |
| 4 | -4.96459 |
| 5 | 2.265444 |
| 6 | 7.23029 |
| 7 | -1.89507 |
| 8 | 0.499542 |
| 9 | -1.56221 |
| 10 | 14.96881 |
| 11 | 6.647786 |
| 12 | 3.282412 |
| 13 | -3.28191 |
| 14 | 1.340886 |
| 15 | -1.18315 |
| 16 | 4.41041 |
| 17 | 8.829762 |
| 18 | 14.38631 |
| 19 | 15.19064 |
| 20 | 2.699908 |
| 21 | -2.31114 |

**Programming in R**

library(readxl)

df\_1 = read\_excel("Autocorrelation\_3.xlsx")

View(df\_1)

ut=c(7,6,-6,-4,3,-4,-5,-8,10,10,6,-4,-4,-7,-2,6,17,24,13,1,1)

Mean=mean(ut)

Mean

N=length(ut)

N

#Calculation of sample auto-covariance and auto-correlation at lag1 and lag2

c0=var(ut)\*((N-1)/N)

c0

c\_k=mat.or.vec(2,1)

r\_k=mat.or.vec(2,1)

for(i in 1:2){

c\_k[i]=(sum((ut[1:(N-i)]-Mean)\*(ut[(1+i):N]-Mean)))/N

r\_k[i]=c\_k[i]/c0

}

c\_k

r\_k

#auto-covariance at lag1 and lag2

cov1=c\_k[1]

cov2=c\_k[2]

#auto-correlation at lag1 and lag2

r1=r\_k[1]

r2=r\_k[2]

#Calculation of coefficients phai1 and phai2

A=array(c(r1,r2),dim=c(2,1))

B=array(c(1,r1,r1,1),dim=c(2,2))

coeff=solve(B)%\*%A

coeff

phai\_1=coeff[1,1]

phai\_1

phai\_2=coeff[2,1]

phai\_2

#Calculation of period of AR(2) process

theta=acos(phai\_1/(2\*sqrt(abs(phai\_2))))

theta

pi=22/7

period=(2\*pi)/theta

period

#Here, in R propgram pi represents 22/7 value

#Calculation of 1st and 2nd term of our fitted AR(2) model

first\_term=phai\_1\*ut[2:20]

second\_term=phai\_2\*ut[1:19]

ut\_des\_est=first\_term+second\_term

ut\_des\_est

ut\_est=ut\_des\_est+Mean

ut\_est

df\_a = append(0,ut\_est)

df\_2 = append(0,df\_a)

View(df\_2)

Data = cbind(df\_1,df\_2)

Data

#using ggplot to plot the graph

library(ggplot2)

ggp = ggplot(NULL, mapping = aes(t)) +

geom\_point(data = Data, mapping = aes(y=ut), col = "black") + geom\_line(data = Data, mapping = aes(y=ut), col = "orange", size = 1) +

geom\_point(data = Data, mapping = aes(y=df\_2), col = "blue") + geom\_line(data = Data, mapping = aes(y=df\_2), col = "green", size = 1) +

labs(

title = paste("Plotting of observed ut and estimated ut against t"),

subtitle = paste("orange\_line=observed ut and green\_line=estimated ut"),

caption = "Data from Model",

x = "t",

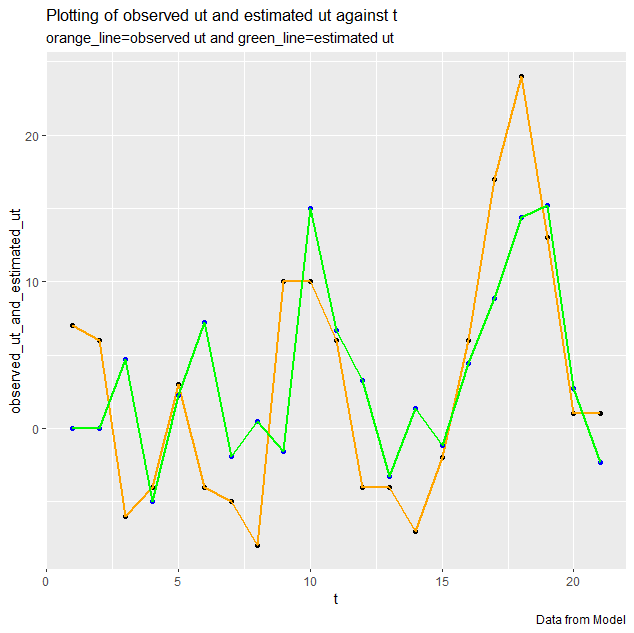
y = "observed\_ut\_and\_estimated\_ut"

)

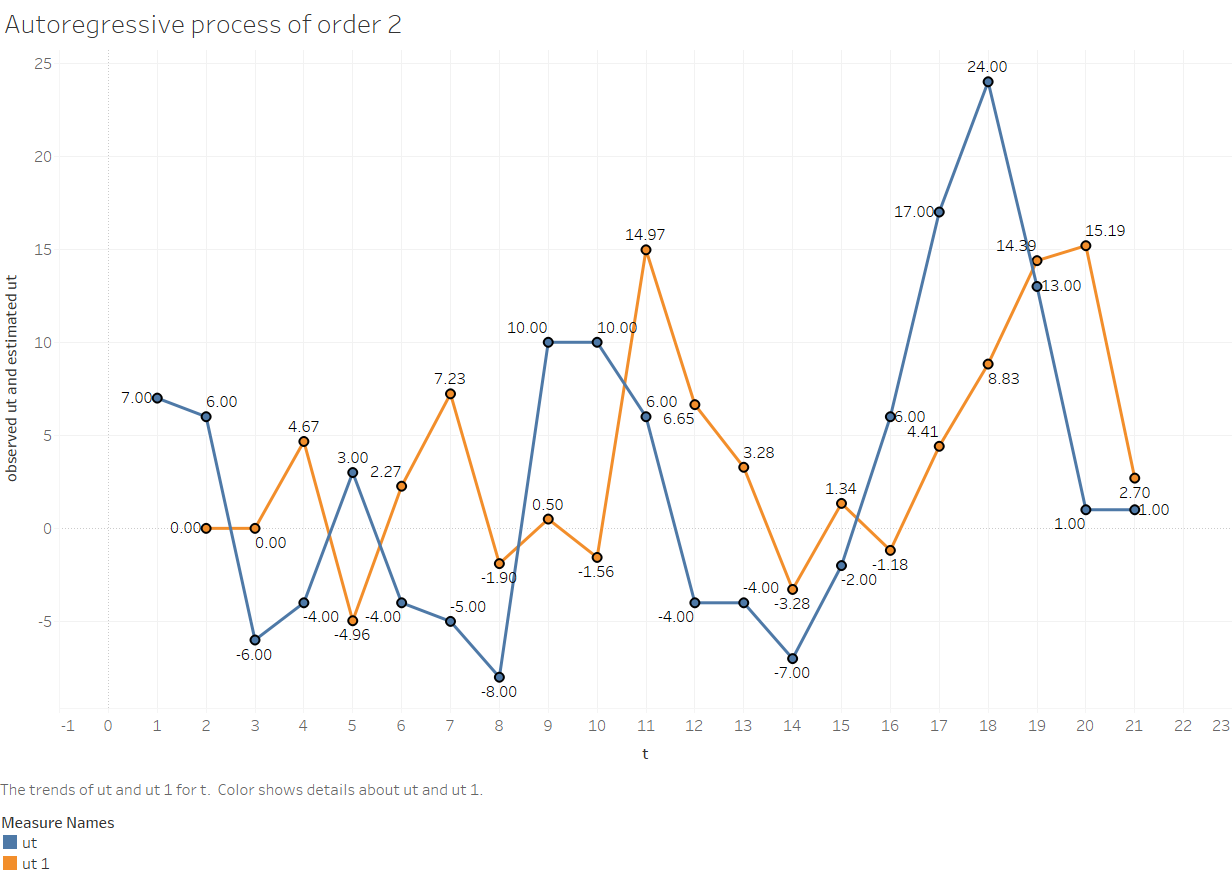
ggp

**Graph plotting**

We plot the graph of observed and estimated values for the second order series considering t along X-axis and **** & **** along Y-axis by Using ggplot2-



**Tableau-**



**Conclusion-**

The fitted AR(2) is-



Hence, The period of the fitted AR series of order 2 is 6.955614.